

# New Analysis of Cumulant Moments in $e^+e^-$ Collisions by SLD Collaboration by Truncated Multiplicity Distributions

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## Abstract

Newly reported normalized cumulant moments of charged particles in  $e^+e^-$  collisions by the SLD collaboration are analyzed by the truncated modified negative binomial distribution (MNBD) and the negative binomial distribution (NBD). Calculated result by the MNBD describes the oscillatory behavior of the data much better than that by the NBD.

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Normalized cumulant moments of negatively charged particles and charged particles obtained from the data in  $e^+e^-$  collisions were analyzed by the truncated modified negative binomial distribution (MNBD) in [1]. Those moments show oscillatory behaviors as the rank of the moment increases, and are well described by the MNBD.

The preliminary data of cumulant moments normalized by factorial moments,  $H_j$  ( $j = 1, 2, \dots$ ) of charged particles reported by the SLD Collaboration at  $\sqrt{s} = 91$  GeV were also analyzed in [1]. The SLD did not explicitly show us the multiplicity distribution. Therefore, we calculated  $H_j$  moments using the observed values of the average charged multiplicity,  $C_2$  moment, and the maximum of the negatively charged particles  $n_{\max} = 27$ . The data were only qualitatively explained by our calculation.

Recently, the new data for the  $H_j$  moments are reported by the SLD [2] and [3]. In this brief report, they are re-analyzed by the MNBD.

The MNBD is given by

$$\begin{aligned}
 P(0) &= \frac{(1+r_1)^N}{(1+r_2)^N}, \\
 P(n) &= \frac{1}{n!} \left(\frac{r_1}{r_2}\right)^N \sum_{j=1}^N {}_N C_j \frac{\Gamma(n+j)}{\Gamma(j)} \left(\frac{r_2-r_1}{r_1}\right)^j \frac{r_2^n}{(1+r_2)^{n+j}}, \quad n = 1, 2, \dots, \quad (1)
 \end{aligned}$$

where  $N$  is a positive integer,  $r_1$  is real, and  $r_2 > 0$ .

At first, Eq.(1) is applied to the multiplicity distribution of negatively charged particles. In order to calculate cumulant moments, factorial moments of charged particles are calculated as

$$\begin{aligned} f_j^{\text{ch}} &= \langle n_{\text{ch}}(n_{\text{ch}} - 1) \cdots (n_{\text{ch}} - j + 1) \rangle \\ &= \sum_n^{n_{\text{max}}} 2n(2n - 1) \cdots (2n - j + 1) P(n), \quad j = 1, 2, \dots, \end{aligned} \quad (2)$$

where  $n_{\text{max}}$  denotes the maximum of the observed negatively charged multiplicity.

The  $j$ -th order normalized cumulant  $K_j$  of charged particles is expressed by the normalized factorial moments  $F_l$  ( $l = 1, 2, \dots$ ) of the charged particles as;

$$\begin{aligned} K_1 &= F_1, \\ K_j &= F_j + \sum_{m=1}^{j-1} j-1 C_{m-1} F_{j-m} K_m, \quad j = 2, 3, \dots, \end{aligned} \quad (3)$$

where

$$F_j = \frac{f_j^{\text{ch}}}{\langle n_{\text{ch}} \rangle^j}.$$

The  $H_j$  moment is defined by

$$H_j = K_j / F_j. \quad (4)$$

The parameters in Eq.(1) are determined by the minimum chi-square ( $\chi_{\text{min}}^2$ ) fit to the observed multiplicity distribution [3] of negatively charged particles with  $n_{\text{max}} = 25$ . The result is shown in Table 1.

The  $H_j$  moment is calculated from Eqs.(1), (2), (3) and (4). The result is shown in Fig.1. Calculation with the MNBD denoted by the solid line well reproduces the oscillatory behavior of the data.

For comparison, we try to fit the new observed multiplicity distribution using the negative binomial distribution (NBD);

$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \left( \frac{\langle n \rangle}{k} \right)^n \left( 1 + \frac{\langle n \rangle}{k} \right)^{-n-k}, \quad n = 0, 1, 2, \dots \quad (5)$$

However, the reasonable  $\chi_{\text{min}}^2$  value cannot be found in the region with  $k > 0$ <sup>1</sup>. See Table 1.

We also analyze the data applying Eq.(1) to the charged particles; only the even terms in Eq.(1) are used with a normalization factor  $C$  [5];

$$f_j^{\text{ch}} = \sum_n^{n_{\text{max}}} 2n(2n - 1) \cdots (2n - j + 1) C P(2n), \quad j = 1, 2, \dots, \quad (6)$$

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<sup>1</sup> The data are analyzed by a weighted superposition of two NBD's [4]

The factor  $C$  is determined by the following equation,

$$C \sum_n^{n_{\max}} P(2n) = 1.$$

Then, the  $H_j$  moment is calculated from Eqs.(6), (3) and (4).

The parameters determined by the  $\chi^2_{\min}$  fit to the multiplicity distribution are also shown in Table 1. The results are depicted in Fig.2. The result obtained from the MNBD, expressed by the solid line, well describes the behavior of the data. However, the result from the NBD, expressed by the dashed line, cannot explain the data.

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## References

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## Table caption

Table 1 The parameters of the MNBD and the NBD used in the analysis of the cumulant moments. The sign "−" and "±" denote negatively charged particles and charged particles, respectively.

## Figure captions

Fig. 1 The normalized cumulant moments  $H_j$  ( $j = 1, 2, \dots$ ) of charged particles in  $e^+e^-$  collisions[3]. The full circles are obtained from the data. The solid line is obtained from the MNBD, which is applied to the negatively charged multiplicity distribution. The calculation based on the NBD is not shown here, because  $\chi^2_{\min}$  value is fairly large and  $k < 0$ .

Fig. 2 The normalized cumulant moments  $H_j$  ( $j = 1, 2, \dots$ ) of charged particles in  $e^+e^-$  collisions[3]. The full circles are obtained from the data. The solid line is obtained from the MNBD, and the dashed line from the NBD. They are applied to the charged multiplicity distribution. The value of  $\chi^2_{\min} = 119.3$  is attributed to the latest data [3].

MNBD	$N$	$r_1$	$r_2$	$\chi^2_{\min}$	
	8	$-0.6873 \pm 0.0027$	$0.6162 \pm 0.0029$	54.1	—
	13	$-0.3580 \pm 0.0054$	$1.249 \pm 0.006$	26.8	$\pm$
NBD		$\langle n \rangle$	$k$	$\chi^2_{\min}$	
		$10.64 \pm 0.02$	$-70.00 \pm 2.57$	1066	—
		$21.01 \pm 0.03$	$24.43 \pm 0.30$	119.3	$\pm$

Table 1

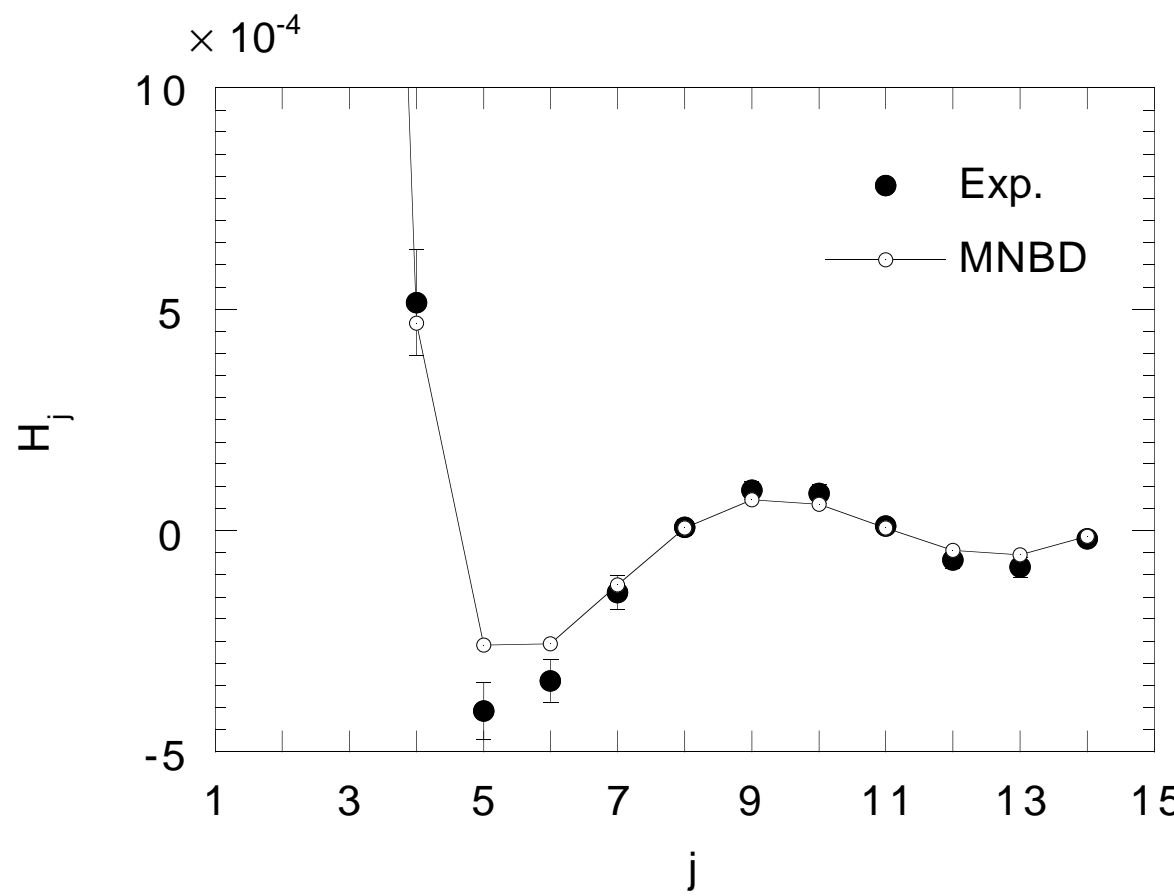


Fig.1

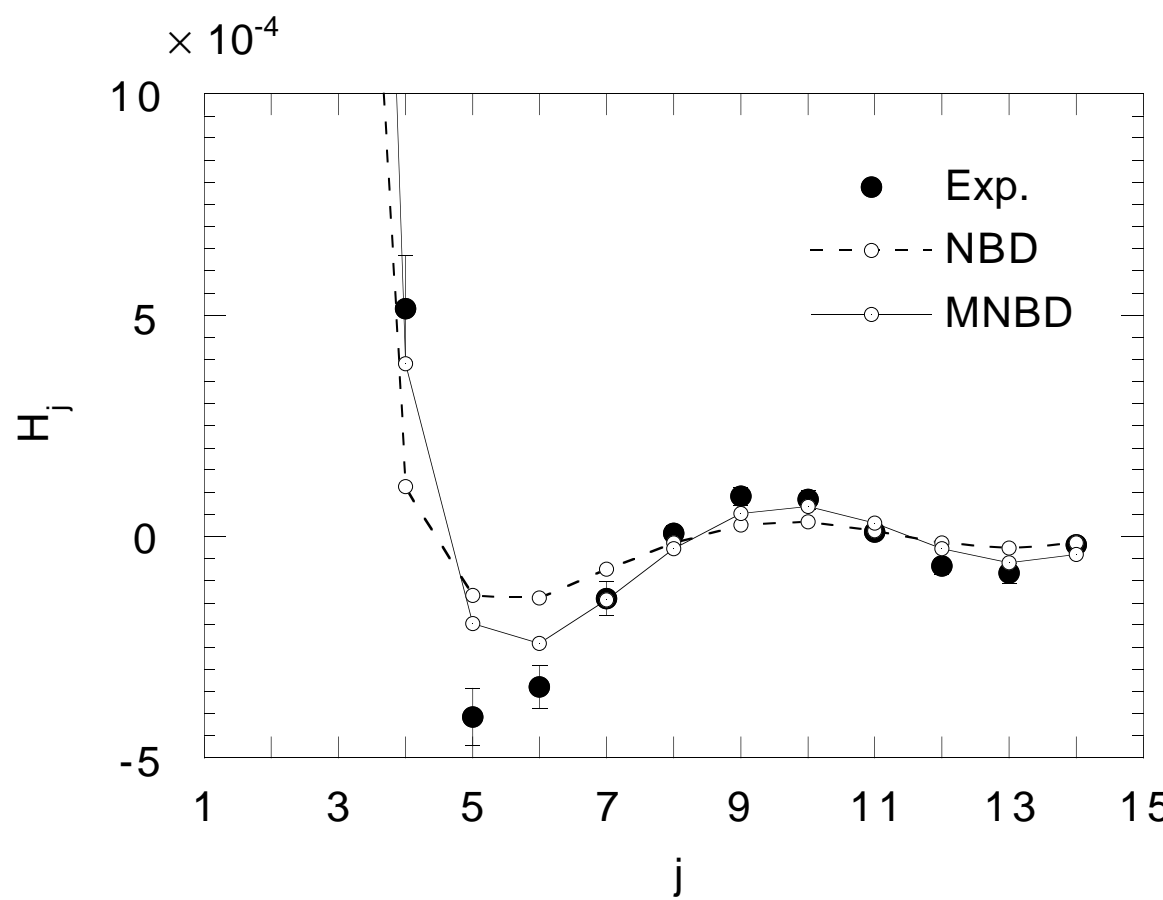


Fig.2